Titolo del corso: Sub-Riemannian geodesics

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Ore frontali di lezione: 15

Periodo di lezione: gennaio-febbraio 2026

Settore/i disciplinare del corso: MAT/05

Tipologia di corso: Avanzato

Modalità di verifica dell'apprendimento: Presentation of a research article related to the subject of the course

Abstract del corso:

We introduce sub-Riemannian manifolds as differentiable manifolds equipped with a bracket generating distribution. A sub-Riemannian distance is naturally defined by considering the curves tangent to the distribution. As in the Riemannian case the metric topology coincides with the manifold topology, and distance minimizers always exist. Through the hamiltonian formalism we build a first order characterization of geodesics, showing that in the sub-Riemannian framework a new class of "abnormal" geodesics exists. The Poisson bracket and the symplectic formalism emerge naturally as a good language for this description. A sub-Riemannian exponential map is introduced. We also present the second order (necessary) Goh condition for singular geodesics.

We treat examples of subRiemannian geodesics with a particular focus on Carnot groups. Finally, (if there is time) we consider some recent research problems about the regularity of sub-Riemannian geodesics and optimal transport in sub-Riemannian setting

Course syllabus:

- Introduction of Hamiltoniano formalism
- Introduction of sub-Riemannian varieties
- Chow-Raschevski and Hopf-Rinow theorems
- Poisson's formalism
- Characterization of normal and abnormal geodesics
- sub-Riemannian exponential map
- Goh conditions
- Geodesics in Heisenberg groups and Carnot groups
- Some regularity results
- Some result on sub-Riemannian transport using properties of geodesic curves